MENIIT
NEET | IIT-JEE | FOUNDATION
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JEE MAINS-2019

10-01-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Mathematics, Chemistry and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above.

PART-A-MATHEMATICS

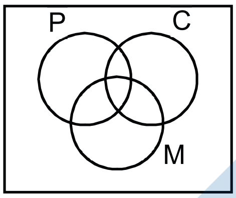
1. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is

(1) 102 (2) 42 (3) 1 (4*) 38

Sol. $n(p) = \left[\frac{140}{5}\right] = 46$

$n(C) = \left[\frac{140}{5}\right] = 28$

$n(M) = \left[\frac{140}{2}\right] = 70$



$N(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap M \cap C)$

$= 46 + 28 + 70 - \left[\frac{140}{15}\right] - \left[\frac{140}{10}\right] - \left[\frac{140}{6}\right] + \left[\frac{140}{30}\right]$

$= 144 - 9 - 14 - 23 + 4 = 102$

So required number of student = 140 – 102 = 38

2. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choices for the ordered triad (a, b, c)?

(1*) (1, 1, 3) (2) $\left(\frac{1}{2}, 2, 0\right)$ (3) $\left(\frac{1}{2}, 2, 3\right)$ (4) (1, 1, 0)

Sol. $y = mx - 4am - 2am^2$

$y = m(x - c) - 2bm - bm^3$

$-4am - 2am^2 = c + ab + bm^2$

(1) $4 + 2m^2 = 3 + 2 + m^2$ $m^2 = 1$ possible

(2) $2 + m^2 = 0 + 4 + 2m^2 \rightarrow$ not possible

(3) $2 + m^2 = 3 + 4 + 2m^2 \rightarrow$ not possible

(4) $4 + 2m^2 = 0 + 2 + m^2 \rightarrow$ not possible

3. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (– 4, 4) at which f is not differentiable. Then S

(1) equal {– 2, – 1, 1, 2} (2) equals {– 2, 2}

(3) is an empty set (4*) equal {– 2, – 1, 0, 1, 2}

Sol. $f(x)=\begin{cases} x & 0\le x<1 \\ x^2 & 1\le x\le 2 \\ -x & -1\le x<0 \\ x^2 & -2\le x<-1 \\ 8-2x & 2<x\le 4 \\ 8+2x & -4\le x<-2 \end{cases}$

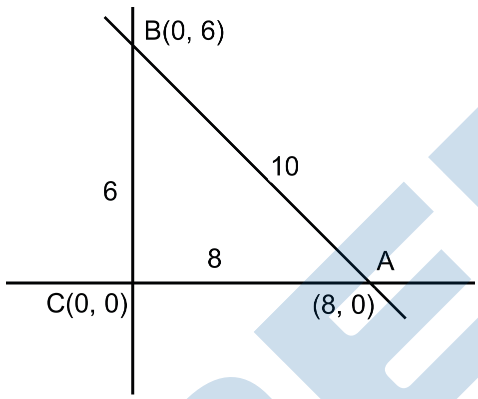
f (x) is not diffential at x = –2, – 1, 0, 1, 2

4. If the line 3x + 4y – 24 = 0 intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

(1*) (2, 2) (2) (4, 4) (3) (3, 4) (4) (4, 3)

Sol. $I=\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1,+by_2+cy_2}{a+b+c}\right)$

$$I=\left(\frac{6(60)+6(0)+10(0)}{24},\frac{10(0)+6(8)+8(0)}{24}\right)=(2,2)$$



5. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is

(1) 1465 (2) 1256 (3*) 1356 (4) 1365

Sol. 16 + 23 + 30 + ...+ 93 = 6 [16 + 93] = 6 × 109 = 654

and 12 + 19 + 26 + ... + 96 = $\frac{13}{2}$ [96 + 12] = 13 × 54 = 702

+ 1356

6. Let f : R → R be a function such that f (x) = $x^3 + x^2$ f '(1) + xf ''(2) + f '''(3), x ∈ R. Then f (2) equals

(1) 30 (2*) – 2 (3) – 4 (4) 8

Sol. f (x) = $x^3 + ax^2 + bx + c$

f '(x) = $3x^2 + 2ax + b$

f ''(x) = 6x + 2a

$f'''(x) = 6a = f'(1) = 3 + 2a + b \Rightarrow a + b = -3$

$b = f'' = 12 + 2a \Rightarrow 2a - b = -12$

$c = f'''(3) \Rightarrow c = 6$ and $a = -5, b = 2$

$\Rightarrow f(x) - x2 + 5x2 + 2x - 6$

$\Rightarrow f(2) = 8 - 20 + 4 + 6 = -2$

7. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t.

Then, $\underset{x \to 1+}{\text{Lim}} \dfrac{(1 - |x| + \sin|1 - x|)\sin\left(\frac{\pi}{2}[1 - x]\right)}{|1 - x|\,[1 - x]}$

(1) equal 1 (2) does not exist (3*) equal 0 (4) equal – 1

Sol. $\lim\limits_{x \to 1+} \dfrac{\left(1 - |x| + \sin|1 - x|\sin\left([1 - x]\frac{\alpha}{2}\right)\right)}{|1 - x|[1 - x]}$

$\lim\limits_{x \to 1^-} \dfrac{\left(1 - x + \sin(x - 1)\sin\left(-\frac{\pi}{2}\right)\right)}{(x - 1)(-1)}$

$\lim\limits_{x \to 1^-} \dfrac{-(x - 1) + \sin(x - 1)}{(x - 1)} = -1 + 1 = 0$

8. Consider the statement: "P(n): $n^2 - n + 41$ is prime. "The which one of the following is true?

(1) P(3) is false but P(5) is true (2) P(5) is false but P(3) is true.

(3*) Both P(3) and P(5) are true (4) Both P(3) and P(5) are false

Sol. $P(n) = n^2 + 41$

$P(3) = 9 - 3 + 41 = 47$

$P(5) = 25 - 5 + 41 = 61$

Hence P(3) and P(5) are both prime

9. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that

$\vec{b} = 2\vec{a}$ and $\vec{a}$ and is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is:

(1) (1, 5, 1) (2) (1, 3, 1) (3) $\left(\frac{1}{2}, 4, -2\right)$ (4*) $\left(-\frac{1}{2}, 4, 0\right)$

Sol. because $b = 2\vec{a}$, so $3 - \lambda_2 = 2\lambda_1$ ....(i)

Because a is perpendicular to c so $6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$.............(ii)

$\Rightarrow (\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$ where $\lambda_1 \in R$

$\Rightarrow \left(-\frac{1}{2},4,0\right)$ satisfied about triplet.

10. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equal

(1) $\frac{1}{3}+e^3$ (2) $-\frac{4}{3}$ (3) $\frac{1}{3}$ (4*) $\frac{1}{3}+e^6$

Sol. $\frac{dy}{dx}+\left(3\sec^2 x\right)y=\sec^2 H$

This is linear differential equation

$\text{integrating factor}=e^{\int 3\sec^2 xdx}=e^{3\tan x}$

Hence $y.e^{\int 3\tan x}.\sec^2 xdx$

$\Rightarrow ye^{3\tan x}=\frac{e^{3\tan x}}{3}+c$

$\Rightarrow y=Ce^{-3\tan x}+\frac{1}{3}$

Giveny $y\left(\frac{\pi}{4}\right)=\frac{4}{3}\Rightarrow\frac{4}{3}=Ce^{-3}+\frac{1}{3}$

$\Rightarrow C = e^3$

Hence $y\left(\frac{\pi}{4}\right)=e^3.e^3+\frac{1}{3}=e^6+\frac{1}{3}$

11. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:

(1*) $\frac{1}{4}$ (2) $4\sqrt{2}$ (3) $\frac{1}{8}$ (4) $2\sqrt{2}$

Sol. third term say $T_3={}^5C_2\left(x^{\log_2 x}\right)^5$

Third term say $T_3={}^5C_2\left(x^{\log_2 x}\right)^5=2560$

$\Rightarrow\left(x^{\log_2 x}\right)^2=256$

Raking logarithm to the base 2 on both sides

$\Rightarrow (\log_2 x)^2 = 8 \Rightarrow (\log_2 x) = \pm 2$

$\Rightarrow x=4,\frac{1}{4}$

Here $x=\frac{1}{4}$

12. If the system of equation

x + y + z = 5

x + 2y + 3z = 9

x + 3y + αz = β

has infinitely many solutions, then $\beta - \alpha$ equals

(1) 21 (2) 5 (3) 18 (4*) 8

Sol. x + y – z = 5

x + 2y + 3z = 9,

x + 3y + αz = β

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 0 \Rightarrow (2\alpha - 9) + (3 - \alpha) + (3 - 2) = 0 \Rightarrow \alpha = 5$$

$$\text{Now, } D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & 0 \end{vmatrix} = 0 \Rightarrow (2\alpha - 9) + (3 - \alpha) + (3 - 2) = 0 \Rightarrow \alpha = 5$$

$\Rightarrow$ at α = 5, b = 13 above 3 planes from common line

13. If $\sum_{i=1}^{20} \left(\frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} \right)^3 = \frac{k}{21}$, then k equal

(1) 200 (2) 400 (3*) 100 (4) 50

Sol. $\sum_{i=1}^{20} \left(\frac{^{20}C_{I-1}}{^{20}C_I + ^{20}C_{I-1}} \right)^3$

Now $\frac{^{20}C_{I-1}}{^{20}C_I + ^{20}C_{I-1}} = \frac{^{20}C_{I-1}}{^{20}C_I} = \frac{I}{21}$

Let given sum be S, so

$$S = \sum_{I=1}^{20} \frac{(i)^3}{21^3} = \frac{1}{(21)^3} \left(\frac{20.21}{2} \right)^2 = \frac{100}{21}$$

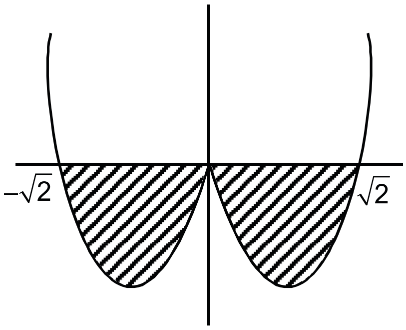
$$\text{given } S = \frac{k}{21} \Rightarrow k = 100$$

14. Let I = $\int_a^b (x^4 - 2x^2)\,dx$. If I is minimum then the ordered pair (a, b) is:

(1) $(\sqrt{2}, -\sqrt{2})$ (2*) $(-\sqrt{2}, \sqrt{2})$ (3) $(-\sqrt{2}, 0)$ (4) $(0, \sqrt{2})$

Sol. $\int_a^b (x^4 - 2x^2q)dx$

Form figure min area is $(-\sqrt{2}, \sqrt{2})$

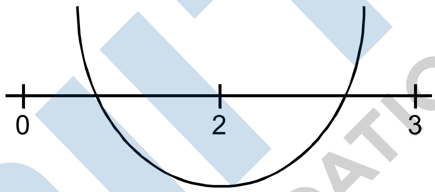


15. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2,3). Then the number of elements in S is

(1*) 11 (2) 10 (3) 12 (4) 18

Sol. Case – 1

$c - 5 > 0$ ....(i)



$f(0) > 0$

$c - 4 > 0$ ....(ii)

$f(2) < 0$

$4(c-5) - 4c + c - 4 < 0$

$c < 24$ ....(iii)

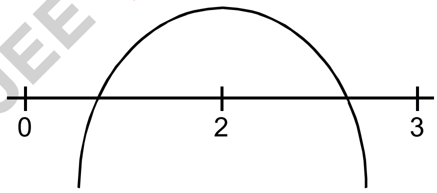
$f(2) > 0$

$9(c) - 5) - 6c - c - 4 > 0$

$4C - 49 > 0 \Rightarrow C > \frac{49}{4}$ ....(iv)

Here (i) $\cap$(ii) $\cap$ (iii) $\cap$(iv)

$C \in \left(\frac{49}{4}, 24\right)$



Case – II

$c - 5 < 0$ ....(i)

$f(0) < 0$

$c < 4$ ....(ii)

$f(2) > 0 \Rightarrow c > 24$ ....(iii)

$f(3) < 0 \Rightarrow c < 49$ ....(iv)

$4 \Rightarrow c \in \phi$

$c \in \left(\frac{49}{4}, 24\right)$

16. If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, – 1), then the radius of C is:

(1) $2\sqrt{5}$ (2) 4 (3) $\sqrt{57}$ (4*) 5

Sol. Tangent at (1, –1) is $x(1) + y(-1) + 2(x + 1) - 3(y - 1) - 12 = 0$

$= 3x - 4y = 7$

Required circle is $(x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$

It pass through (4, 0)

$\Rightarrow 9 + 1 + \lambda(12 - 7) = 0 \Rightarrow \lambda = -2$

$\Rightarrow$ required circle is $x^2 + y^2 - 8x + 10y + 16 = 0$

$\Rightarrow$ Radius $= \sqrt{16 + 25 - 16} = 5$

17. A point P moves on the line $2x - 3y + 4 = 0$. If Q (1, 4) and R (3, – 2) are fixed points, then the locus of the centroid of ΔPQR is a line :

(1*) with slope $\frac{2}{3}$ (2) parallel to y-axis (3) parallel to x-axis (4) with slope $\frac{3}{2}$

Sol. Let point P is (α, β) and centroid of ΔPQR is (h, k), then $3h = \alpha + 1 + 3$ and

$3k = \beta + 4 - 2$

$\Rightarrow \alpha = 3h - 4$ and $\beta = 3k - 2$

Because (α, β) lies on $2x - 3y + 4 = 0$

$\Rightarrow 2(3h - 4) - 3(k - 2) + 4 = 0$

$\Rightarrow$ locus is $6x - 9y + 2 = 0$ whose slope is $\frac{2}{3}$

18. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to

(1) $\frac{3}{4}$ (2) $\frac{3}{2}$ (3) $\frac{5}{4}$ (4*) $\frac{7}{4}$

Sol. $5, 5r, 5r^2$ sides of triangle,

$5 + 5r > 5r^2$ ………………(1)

$5 + 5r^2 > 5r$ ………………(2)

$5r + 5r^2 > 5$ ………………(3)

From $r^2 - r - 1 < 0$

$\left[r - \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)\right]$ ………..(4)

From (2)

$r^2 - r + 1 > 0 \Rightarrow r \in R$ ……….(5)

From (3)

$r^2 - r + 1 > 0$

So, $\left(r + \frac{\sqrt{1+\sqrt{5}}}{2}\right)\left(r + \frac{1-\sqrt{5}}{2}\right) > 0$

$r \in \left(-\infty, \frac{1+\sqrt{5}}{2}\right) \cup \left(-\frac{1-\sqrt{5}}{2}, \infty\right)$ ....(6)

From (4), (5), (6) $r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$

Now check options

19. Let $d \in R$ and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is:

(1) – 7 (2) $2(\sqrt{2}+1)$ (3) $2(\sqrt{2}+2)$ (4*) – 5

Sol. $|A| = \begin{vmatrix} -2 & 4+d & (\sin\theta-2) \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta) & (-\sin\theta)+2+2d \end{vmatrix}$

$= \begin{vmatrix} -2 & 4+d & (\sin\theta-2) \\ 1 & (\sin\theta) & d \\ 1 & 0 & 0 \end{vmatrix}$ (New $R_3 = R_3 - 2R_2 + R_1$)

$=(4 + d)d - \sin^2\theta + 4 = (d + 2)^2 - \sin^2\theta$

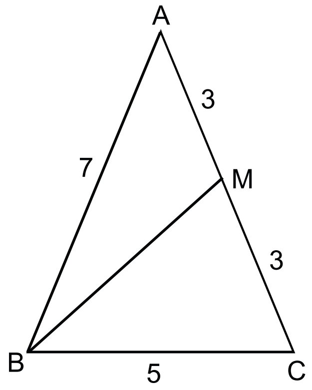
Because minimum value of $|A| = 8 \Rightarrow (d + 2)^2 = 9 \Rightarrow d = 1$ or -5

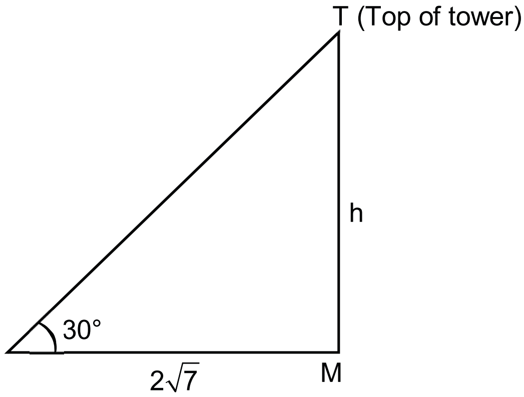
20. Consider a triangular plot ABC with sides AB = 7 m, BC = 5 m and CA = 6 m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is :

(1) $7\sqrt{3}$ (2*) $\frac{2}{3}\sqrt{21}$ (3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

Sol. Length of median $BM = \frac{1}{2}\sqrt{2(BC^2 + BA^2) - (AC)^2} = \frac{1}{2}\sqrt{2(25+49)-36}$

$= \frac{1}{2}\sqrt{112} = \sqrt{\frac{112}{4}} = \sqrt{28} = 2\sqrt{7}$





Let h be height of tower, given tan 30° $= \frac{h}{2\sqrt{h}} \Rightarrow h = 2\sqrt{\frac{7}{3}} = \sqrt{\frac{28}{3}}$

21. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is

(1*) $\frac{19}{72}$ (2) $\frac{15}{72}$ (3) $\frac{19}{36}$ (4) $\frac{13}{36}$

Sol. P(7 or 8)

= P (H) P(7 or 8) + P(T) P(7 or 8) = $= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{11}{22} + \frac{1}{9} = \frac{19}{72}$

22. Let A be a point on the line $\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$ and B (3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane x – 4y + 3z = 1 is

(1) $\frac{1}{8}$ (2) $\frac{-1}{4}$ (3) $\frac{1}{2}$ (4*) $\frac{1}{4}$

Sol. Let A is (1– 3μ, μ –1, 2 + 5μ)

$\overline{AB}$ = (3μ + 2)i + (3 – μ) j + (4 – 5μ)

$\hat{k}$ which is parallel to plane x – 4y + 3z = 1

$\Rightarrow$ 1(3μ + 2) – 4(3 – μ) + 3(4 – 5μ) = 0

$= -8\mu + 2 = 0 \Rightarrow \mu = \frac{1}{4}$

23. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is

(1) 10 : 3 (2) 5 : 8 (3*) 4 : 9 (4) 6 : 7

Sol. $\mu = \frac{1+3+8+x+y}{5}$

$25 = 12 + x + y \Rightarrow x + y = 13$ ....(1)

$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$

$9.2 = \frac{1+9+64+x^2+y^2}{5} - 25$

$34.2 \times 5 = 74 + x^2 + y^2$

$171 = 74 + x^2 + 2xy$

$97 = x^2 + y^2$ ....(2)

$(x + y)^2 = x^2 + y^2 + 2xy$

$169 - 97 = 2xy \Rightarrow xy = 36$

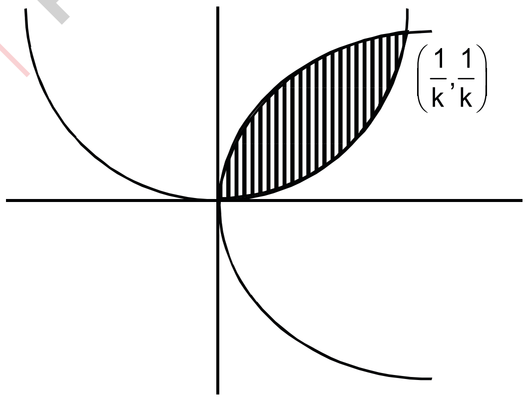
T = 4, 9

So ration is $\frac{4}{9}$ or $\frac{9}{4}$

24. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is

(1*) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\sqrt{3}$ (4) $\frac{\sqrt{3}}{2}$

Sol. $y = kx^2$, $x = ky^2$



$\Rightarrow x = k(k^2x^2) \Rightarrow x = 0$ or $x^3 = \left(\frac{1}{k}\right)^2 \Rightarrow x = \frac{1}{k}, 0$

Point of intersection are $\left(\frac{1}{k}, \frac{1}{k}\right)$ and (0, 0)

$\text{Area} = \int_0^{1/k}\left(\sqrt{\frac{x}{k}} - kx^1\right)dx = 1 \Rightarrow \left(\frac{1}{\sqrt{k}}\frac{x^{3/2}}{3/2} - \frac{kx^3}{3}\right)^{1/k}$

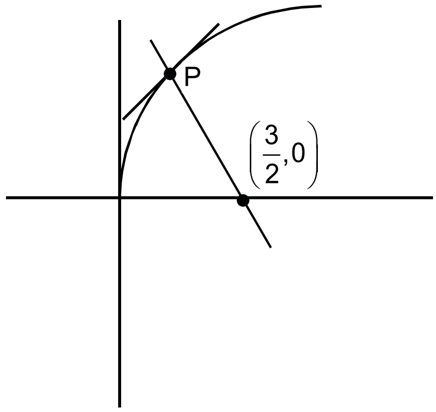
$= 1 \Rightarrow \frac{2}{3k^2} = 1 \Rightarrow k^2 = \frac{1}{3}$

$k = \frac{1}{\sqrt{3}}$

25. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}$, $(x > 0)$, is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{5}{4}$ (3*) $\frac{\sqrt{5}}{2}$ (4) $\frac{3}{2}$

Sol. Let P be the point nearest to $\left(\frac{3}{2}, 0\right)$, then normal at P will pass through $\left(\frac{3}{2},0\right)$,



Let Co – ordinates of P be $\left(\frac{t^2}{4}.\frac{t}{2}\right)$

Hence equation of normal is $y+tx=\frac{t}{2}+\frac{t^2}{4}$

The line passes through $\left(\frac{3}{2},0\right)$

$\frac{3t}{2}=\frac{t}{2}+\frac{t^2}{4}\Rightarrow t=2$ (–2, 0 are rejected)

hence nearest point is (1, 1)

distacne $\sqrt{\left(\frac{3}{2}-1\right)^2+(1-0)^2}=\frac{\sqrt{5}}{2}$

26. The plane passing through the point (4, – 1, 2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point

(1) (– 1, – 1, – 1) (2) (1, 1, – 1) (3*) (1, 1, 1) (4) (– 1, – 1, 1)

Sol. $\vec{n}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 3\\ 1 & 2 & 3\end{vmatrix}=7\hat{i}-7\hat{j}+7\hat{k}$

Equation of place is – 7(x – 4) –7(y + 1) +7(z – 2) = 0

$\Rightarrow -7x-7y+7z+7=0\Rightarrow x+y-z=1$

27. The sum of all values of $\theta\in\left(0,\frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$ is

(1) π (2*) $\frac{\pi}{2}$ (3) $\frac{3\pi}{8}$ (4)

Sol. $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$

Let $\cos^2 2\theta = t$

$\Rightarrow 1-\cos^2 2\theta+\cos^4 2\theta=\frac{3}{4}$

$\Rightarrow t=\frac{1}{2}\Rightarrow \cos^2 2\theta=\frac{1}{2}$

$\Rightarrow 2\cos^2 2\theta - 1 = 0 \Rightarrow \cos 4\theta = 0$

$$\Rightarrow 40 = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{8} \Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \in \left[0, \frac{\pi}{2}\right]$$

Sum of values of θ is $\frac{\pi}{2}$

28. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is

(1*) $x - y + 1 = 0$ (2) $x - y - 3 = 0$ (3) $x - y + 7 = 0$ (4) $x - y + 9 = 0$

Sol. Hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1$

Equation of its tangent in slope from is $y = mx \pm \sqrt{5m^2 - 4}$

Hence tangent with slope 1 is $y = x \pm 1$

29. Let z_1 and z_2 be two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then

(1) $\text{Re}(z) = 0$ (2) $|z| = \sqrt{\frac{5}{2}}$ (3) $\text{Im}(z) = 0$ (4) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$

Ans. No option is correct

Sol. $\left|\frac{3z_1}{2z_2}\right| = 2$

Let $\frac{3z_1}{3z_2} = 2\cos\theta + 2(\sin\theta)i$

$$\Rightarrow \frac{2z_1}{3z_2} = \frac{1}{2}\cos\theta - \frac{1}{2}(\sin\theta)$$

Given, $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = \frac{5}{2}\cos\theta + \frac{3}{2}(\sin\theta)i$

Which is neither purely real nor purely imaginary and $|z|$ depends on θ.

30. Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}} \cos\theta}{\sin^{n+1}\theta} d\theta$. is equal to

(where C is a constant integration)

(1) $\frac{n}{n^2+1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$ (2) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$ (4*) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

Sol. $\int\frac{(\sin^n\theta-\sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$

$=\int\frac{(t^n-t)^n\,dt}{t^{n+1}}$ (Put $\sin\theta = t$)

$=\int\frac{t\left(1-\frac{1}{t^{n-1}}\right)}{t^{n+1}}dt=\int\frac{\left(1-\frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n}dt$

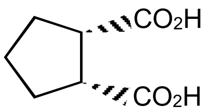
Put $1-\frac{1}{t^{n-1}}=z\Rightarrow\frac{(n-1)}{t^n}dt=dz,$

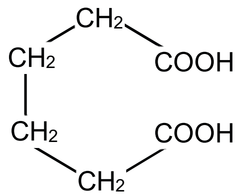
$\Rightarrow I=\frac{1}{n-1}\int z^{\frac{1}{n}}dz=\frac{z^{\frac{1}{n}+1}}{\left(\frac{1}{n}+1\right)(n-1)}+c=\frac{n\left(1-t^{1-n}\right)^{n+1}}{n^2-1}+c$

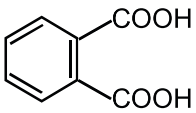
$=\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{h+1}{n}}+c$

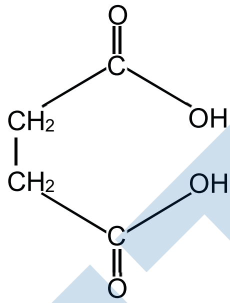
PART-B-CHEMISTRY

31. Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride?

(1) 

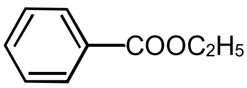
(2*) 

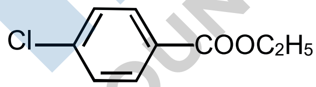
(3) 

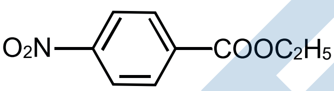
(4) 

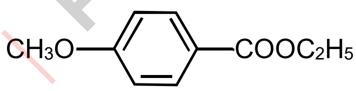
Sol. Adipic acid will give unstable 7 membered anhydride.

32. The decreasing order of ease of alkaline hydrolysis for the following esters is -

(I) 

(II) 

(III) 

(IV) 

(1) III > II > IV > I (2*) III > II > I > IV (3) IV > II > III > I (4) II > III > I > IV

Sol. Electron withdrawing group enhances the rate of hydrolysis.

33. Hall-Heroult's process is given by:

(1) $ZnO + C \xrightarrow{\text{Coke,1673K}} Zn + CO$

(2) $Cu^{2+}(aq) + H_2(g) \rightarrow Cu(s) + 2H^+(aq)$

(3*) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(4) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

Sol. Hall-Herolut's process is given by

$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$

$2Al_2O_3(\ell) \rightleftharpoons 4Al^{3+}(\ell) + 6O^{2-}(\ell)$

At cathode: $4Al^{3+}(\ell) + 12e^- \longrightarrow 4Al(\ell)$

At anode: $6O^{2-}(\ell) \longrightarrow 3O_2(g) + 12e^-$

$2C + 3O_2 \longrightarrow 3CO_2(\uparrow)$

34. A process has $\Delta H = 200\ J\ mol^{-1}$ and $\Delta S = 40\ JK^{-1}\ mol^{-1}$. Out of the values given below, choose the minimum temperature above which the process will be spontaneous:

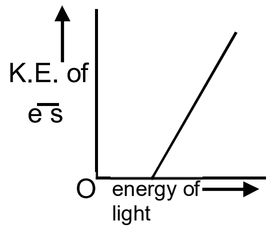
(1*) 5 K (2) 4 K (3) 20 K (4) 12 K

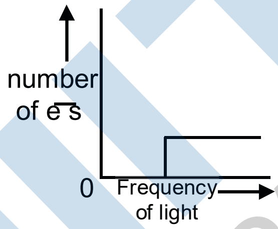
Sol. For spontaneous reaction $\Delta G < 0$

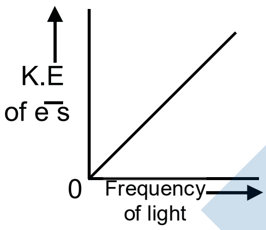
$$\Delta H - T\Delta S < 0$$

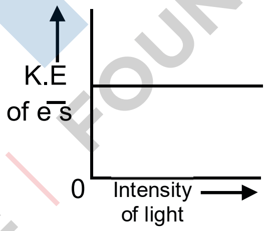
$$T > \frac{\Delta H}{\Delta S} = 5K$$

35. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected from metal surface?

(1)


(2)


(3*)


(4)


Sol. $\because (KE)_{e^-} = h.\nu - \varphi$

so, the graph between KE and will be a straight line having positive slope and negative intercept on y-axis.

36. The electronegativity of aluminium is similar to :

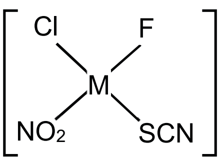
(1*) Beryllium (2) Boron (3) Lithium (4) Carbon

Sol. Diagonal relationship.

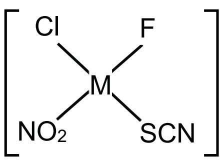
37. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is :

(1) 16 (2) 4 (3) 8 (4*) 12

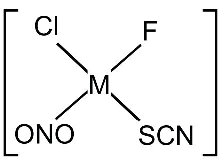
Sol.



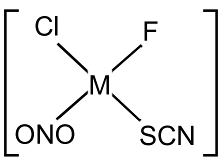
= 3 Gi



=3 Gi



=3 Gi



=3 Gi

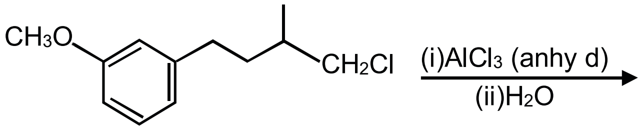
Total = 12 GI

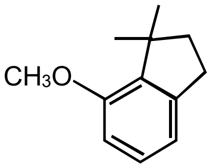
38. The metal used for making X-ray tube window is:

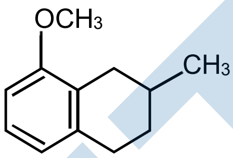
(1) Na (2*) Be (3) Ca (4) Mg

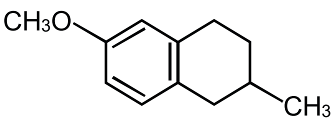
Sol. Fact based

39. The major product of the following reaction is:

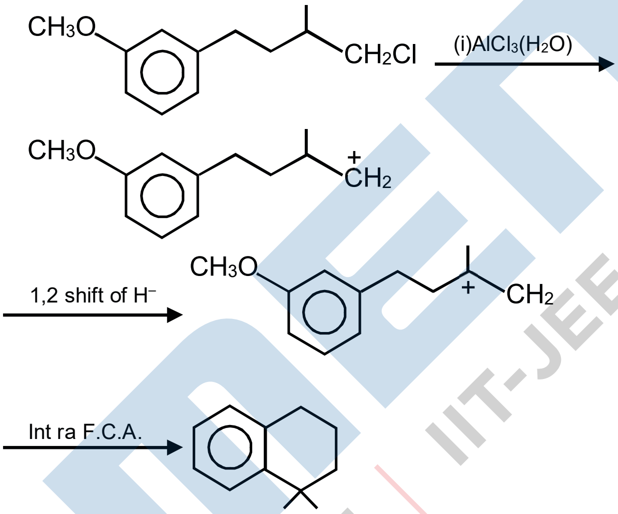


(1)


(2)


(3)


(4)


Sol.


40. Two pi and half sigma bonds are present in:

(1*) N_2^+ (2) O_2^+ (3) O_2 (4) N_2

Sol. $n_2^{\oplus} \Rightarrow BO = 2.5 \Rightarrow \left[\pi - \text{Bond} = 2 \,\&\, \sigma - \text{Bond} = \frac{1}{2}\right]$

$N_2 \Rightarrow$ B. O.$\Rightarrow 2.5 \Rightarrow [\pi$–Bond = 2 & σ–Bond = 1]

$O_2^{\oplus}$ = B.O. $\Rightarrow 2.5 \Rightarrow [\pi$ –bond = 1.5 & σ –Bond = 1]

$O_2 \Rightarrow$ B.O. $\Rightarrow 2 \Rightarrow [\pi$–Bond $\Rightarrow$ 1 & σ –Bond = 1]

41. The values of K_P/K_C for the following reactions at 300 K are, respectively:

[At 300 K, RT = 24.62 dm^3 atm mol^{-1}]

$N_2(g) + O_2(g) \rightleftharpoons 2\,NO(g)$

$N_2O_4(g) \rightleftharpoons 2\,NO_2(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2\,NH_3(g)$

(1) 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}, 1.65 ×10^{-3} dm^{-6} atm^{-2} mol^2

(2) 1, 4.1 × 10^{-2} dm^{-3} atm^{-1} mol, 606 dm^6 atm^2 mol^{-2}

(3) 1, 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}

(4*) 1, 24.62 dm^3 atm mol^{-1}, 1.65 × 10^{-3} dm^{-6} atm^{-2} mol^2

Sol. $\because \frac{K_P}{K_C} = (RT)^{\Delta n_g}$

42. Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is:

(1) A is more polluted than B

(2*) B is more polluted than A

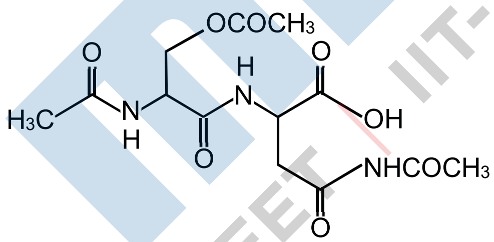
(3) A is suitable for drinking, whereas B is not

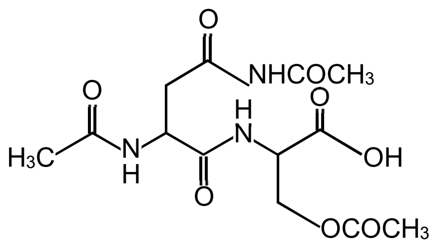
(4) Both A and B are suitable for drinking

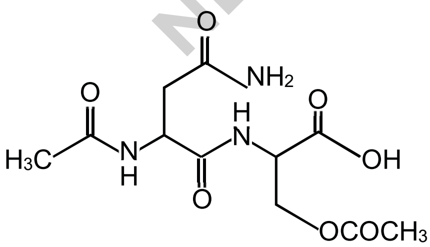
Sol. More polluted water has high biological oxygen demand.

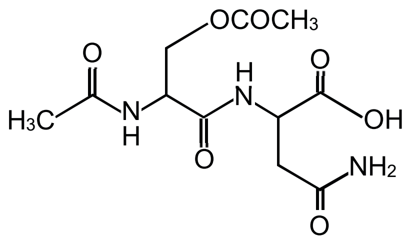
43. The correct structure of product 'P' in the following reaction is:

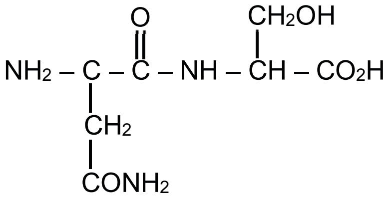
$$\text{Asn} - \text{Ser} + \underset{(\text{excess})}{(CH_3CO)_2O} \xrightarrow{NEt_3} P$$

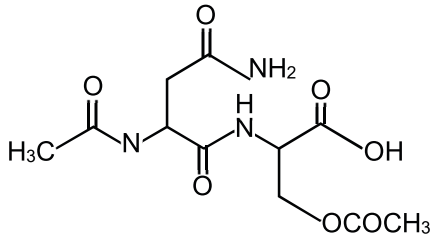
(1) 

(2) 

(3*) 

(4) 

Sol. 

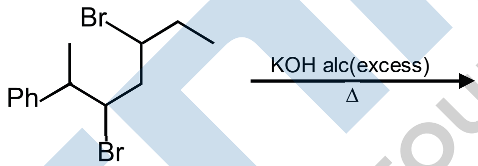


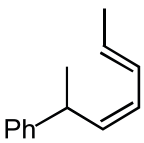
44. Which premitive unit cell has unequal edge lengths ($a \neq b \neq c$) and all axial angles different from 90°?

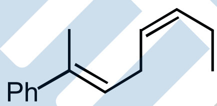
(1*) Triclinic (2) Tetragonal (3) Monoclinic (4) Hexagonal

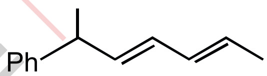
Sol. Theory based

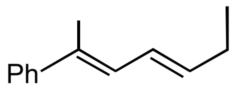
45. The major product of the following reaction is:

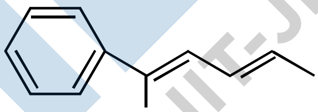


(1) 

(2) 

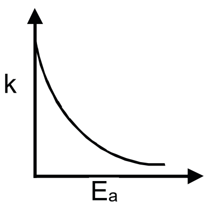
(3) 

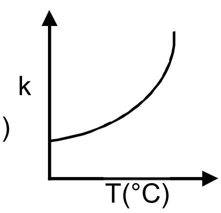
(4*) 

Sol. Major product will be  Due to E_2 elimination & extended

conjugatin

46. Consider the given plots for a reaction obeying Arrhenius equation (0°C < T < 300°C) : (k and E_a are rate constant and activation energy respectively)

(I) 

(II) 

Choose the correct option:

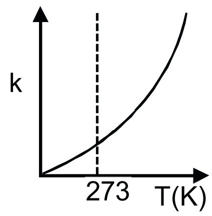
(1) Both I and II are wrong (2) I is wrong but II is right

(3) I is right but II is wrong (4*) Both I and II are correct

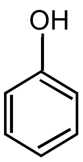
Sol. $\because K = A.e^{-\frac{E_a}{RT}}$

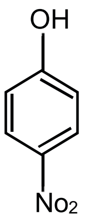
where T in Kelvin

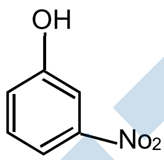
so

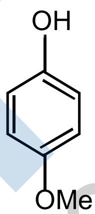


47. The increasing order of the pKa values of the following compounds is:

(A) 

(B) 

(C) 

(D) 

(1) D < A < C < B (2*) B < C < A < D (3) B < C < D < A (4) C < B < A < D

Sol. Higher the Ka value lower the pKa value.

48. Consider the following reduction processes:

$Zn^{2+} + 2e^- \rightarrow Zn(s)$; E° = – 0.76 V

$Ca^{2+} + 2e^- \rightarrow Ca(s)$; E° = – 2.87 V

$Mg^{2+} + 2e^- \rightarrow Mg(s)$; E° = – 2.36 V

$Ni^{2+} + 2e^- \rightarrow Ni(s)$; E° = – 0.25 V

The reducing power of the metals increases in the order:

(1) Ca < Mg < Zn < Ni (2*) Ni < Zn < Mg < Ca (3) Ca < Zn < Mg < Ni (4) Zn < Mg < Ni < Ca

Sol. Greater SOP $\Rightarrow$ More tendency to be oxidised

$\Rightarrow$ More reducing power

49. Which of the following is not an example of heterogeneous catalytic reaction?

(1) Combustion of coal (2) Haber's process

(3) Ostwald's process (4) Hydrogenation of vegetable oils

Sol. Fact based.

50. Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light?

$$\underset{\delta}{CH_3} - \underset{\gamma}{CH_2} - \underset{\beta}{CH} = \underset{\alpha}{CH_2}$$

(E)

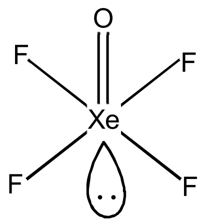
(1*) γ-hydrogen (2) α-hydrogen (3) β-hydrogen (4) δ-hydrogen

Sol. Allylic radical will form in presence of sunlight.

51. The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$ respectively, are:

(1) sp^3d and 1 (2*) sp^3d^2 and 1 (3) sp^3d^2 and 2 (4) sp^3d and 2

Sol.



52. If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct ?

(1*) DCM and H_2O would stay as lower and upper layer respectively in the S.F.

(2) DCM and H_2O will be miscible clearly.

(3) DCM and H_2O will make turbid / colloidal mixture

(4) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)

Sol. Density of DCM is higher than H_2O.

53. Wilkinson catalyst is :

(1) $[(Et_3P)_3RhCl]$ (2*) $[(Ph_3P)_3RhCl]$ ($Et=C_2H_5$)

(3) $[(Ph_3P)_3IrCl]$ (4) $[(Et_3P)_3IrCl]$

Sol. Fact based

54. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are:

(1) 2 and 0 (2) 3 and 2 (3*) 3 and 1 (4) 2 and 1

Sol. Fact based

55. The chemical nature of hydrogen peroxide is:

(1) Reducing agent in basic medium, but not in acidic medium

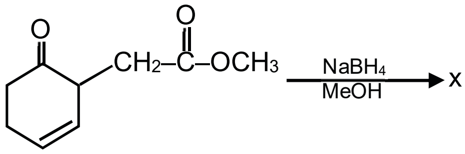
(2*) Oxidising and reducing agent in both acidic and basic medium

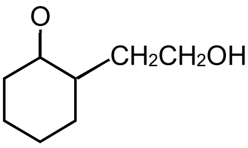
(3) Oxidising agent in acidic medium, but not in basic medium

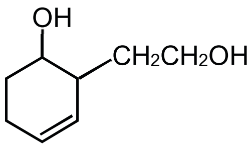
(4) Oxidising and reducing agent in acidic medium, but not in basic medium

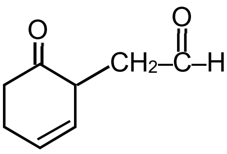
Sol. Fact based.

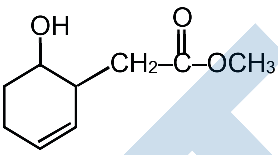
56. The major product 'X' formed in the following reaction is :



(1) 

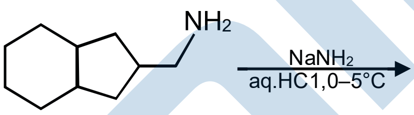
(2) 

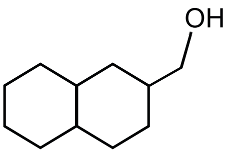
(3) 

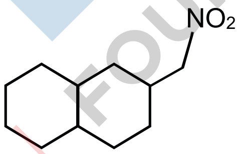
(4*) 

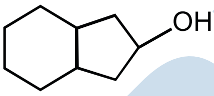
Sol. $NaBH_4$ don't reduce ester.

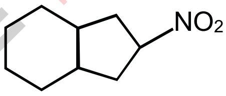
57. The major product formed in the reaction given below will be:



(1) 

(2) 

(3) 

(4) 

Ans. Bonus

Sol. Answer should be 

58. Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vapour pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapour in equilibrium with a solution containing 40 mole percent of A at this temperature is:

(1) $x_A = 0.4$; $x_B = 0.6$

(2) $x_A = 0.76$; $x_B = 0.24$

(3) $x_A = 0.37$; $x_B = 0.63$

(4*) $x_A = 0.28$; $x_B = 0.72$

Sol. $P_{solution} = 7 \times 10^3 (0.4) + 12 \times 10^3 (0.6)$

$= 10^4$ Pa

In vapour phase,

$x_A = \frac{7 \times 10^3 \times 0.4}{10^4} = 0.28$

$x_B = 1 - x_A = 0.72$

59. A mixture of 100 m mol of $Ca(OH)_2$ and 2 g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are :

(Molar mass of $Ca(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1}, respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

(1) 13.6 g, 0.14 mol L^{-1}
(2) 1.9 g, 0.14 mol L^{-1}
(3) 13.6 g, 0.28 mol L^{-1}
(4*) 1.9 g, 0.28 mol L^{-1}

Sol. $Ca(OH)_2 + Na_2SO_4 \rightarrow CaSO_4 + 2NaOH$

0.1 mole $\quad \frac{2}{143}$ mole

$$LR \rightarrow \frac{2}{143}\text{mole} \quad 2\left(\frac{2}{143}\right)\text{mole}$$

Mass of $CaSO_4 = \frac{2}{143} \times 136 = 1.9$ gram

$$\left[OH^-\right] = \frac{2\left(\frac{2}{143}\right)}{100/1000} = 0.28 \text{ Mol / L}$$

60. The effect of lanthanoid contraction in the lanthanoid series of elements by and large means:

(1) increase in both atomic and ionic radii

(2) increase in atomic radii and decrease in ionic radii

(3) decrease in atomic radii and increase in ionic radii

(4*) decrease in both atomic and ionic radii

Sol. Atomic & ionic radii decreases

PART-C-PHYSICS

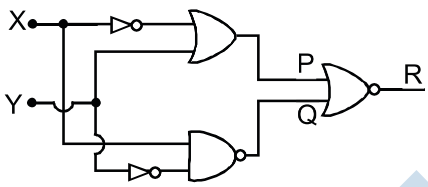
61. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s).

$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is:

(1) 4×10^4 N/C 2) 4.5×10^4 N/C (3*) 3×10^4 N/C (4) 6×10^4 N/C

Sol. E = CB

62. To get output '1' at R, for the given logic gate circuit the input values must be:



(1*) X = 1, Y = 0 (2) X = 0, Y = 1 (3) X = 0, Y = 0 (4) X = 1, Y = 1

Sol. $R = \overline{P + Q} = \overline{(\overline{x} + y) + (\overline{x\overline{y}})}$

$= \left(\overline{\overline{x} + y}\right) \cdot (x\overline{y})$

$= (x \cdot \overline{y}) \cdot (x\overline{y})$

$= x\overline{y}$

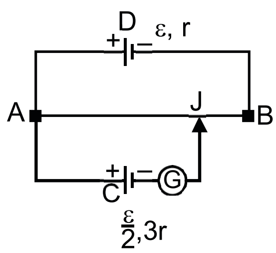
63. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is:

(1) 2.9 cm (2) 1.7 cm (3*) 5.1 cm (4) 4 cm

Sol. $\frac{dV}{dt} = \phi - a\sqrt{2gh} = 0(\text{for maximum height})$

$h = \frac{\phi^2}{2ga^2} = \frac{10^{-8}}{2 \times 9.8 \times 10^{-8}} = 5.1\,\text{cm}$

64. A potentiometer wire AB having length L and resistance 12r is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\varepsilon/2$ and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is:



(1) $\frac{11}{12}L$ (2) $\frac{5}{12}L$ (3) $\frac{11}{24}L$ (4*) $\frac{13}{24}L$

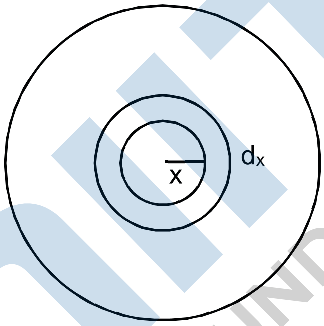
Sol. $V_{AJ} = IR_{AJ}$

$$\frac{E}{3} = \left(\frac{E}{12r+3r}\right) \times \left(\frac{x}{L} \times 12r\right)$$

65. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is :

(1) μFR / 2 (2*) $\frac{2}{3}\mu FR$ (3) μFR / 6 (4) μFR / 3

Sol. $d\tau = (\mu dN)x$



$$= \mu\left(\frac{F}{\pi R^2} \times 2\pi \times dr\right)x$$

$$\tau = \int_0^R d\tau$$

66. A heat source at T = 10^3 K is connected to another heat reservoir at T = 10^2 K by a copper slab which is 1m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is :

(1) 120 Wm^{-2} (2) 200 Wm^{-2} (3*) 90 Wm^{-2} (4) 65 Wm^{-2}

Sol. $$\frac{\Delta Q}{\Delta t} = \frac{kA}{\ell}(T_2 - T_1)$$

$$\frac{1}{A}\left(\frac{\Delta Q}{\Delta t}\right) = \frac{k}{\ell}(T_2 - T_1)$$

67. An insulating thin rod of length *l* has a linear charge density $\rho(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is:

(1) $n\rho\ell^3$ (2) $\frac{\pi}{3}n\rho\ell^3$ (3) $\pi n\rho\ell^3$ (4*) $\frac{\pi}{4}n\rho\ell^3$

Sol. dM = di A

$$= \left(\frac{dq\omega}{2\pi}\right)\pi x^2$$

$$= (\rho dx)\frac{\omega}{2\pi}\pi x^2$$

$$M = \int_0^L dM$$

68. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to :

(1) 500 keV (2) 100 keV (3) 1 keV (4*) 25 keV

Sol. $k = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$

$$= \frac{\left(6.62\times10^{-34}\right)^2}{2\times9.1\times10^{-31}\times\left(7.5\times10^{-12}\right)^2}\times\frac{1}{1.6\times10^{-19}}\text{eV}$$

Alternate method

$$\lambda = \sqrt{\frac{150}{V}}$$

$$\Rightarrow 7.5\times10^{-12} = \sqrt{\frac{150}{V}}$$

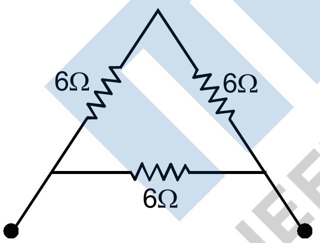
$$V = \frac{80}{3}\text{kV}$$

$$\text{Energy} = \frac{80}{3}\text{keV} \approx 25\text{ keV}$$

69. A uniform metallic wire has a resistance of 18Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is :

(1*) 4Ω (2) 2Ω (3) 8Ω (4) 12Ω

Sol.



$R_{net} = 4\Omega$

70. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is :

(1*) mv^2 (2) $\frac{3}{2}mv^2$ (3) $2mv^2$ (4) $\frac{1}{2}mv^2$

Sol. initially, kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM_e}{r}$

By conservation of M.E.

$$\frac{-GM_em}{r}+KE=0$$

$$KE=\frac{GM_em}{r}=mv^2$$

71. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be:

(1) $\frac{Q}{12\pi\in_0}\frac{ab+bc+ca}{abc}$ (2) $\frac{Q(a^2+b^2+c^2)}{4\pi\in_0(a^3+b^3+c^3)}$ (3) $\frac{Q}{4\pi\in_0(a+b+c)}$ (4*) $\frac{Q(a+b+c)}{4\pi\in_0(a^2+b^2+c^2)}$

Sol. $Q_1 + q_2 + Q_3 = Q$ ....(1)

$$\frac{Q_1}{4\pi a^2}=\frac{Q_2}{4\pi b^2}=\frac{Q_3}{4\pi c^2}=k \text{ ....(2)}$$

Subs. Q_1, Q_2, Q_3 in (1)

$$k=\frac{Q}{4\pi\left(a^2+b^2+c^2\right)}$$

$$v=\frac{kQ_1}{a}+\frac{kQ_2}{b}+\frac{kQ_3}{c}$$

72. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is :

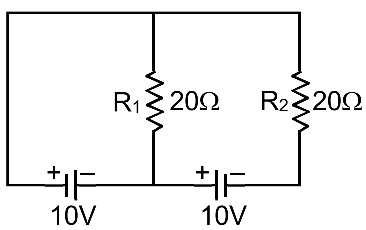
(1) 16 (2*) 40 (3) 410 (4) 640

Sol. $\frac{128\text{ kg}}{m^3}=\frac{64\ (2\text{kg})}{10\text{ cm}^3}$

$$=\frac{64(2\text{ kg})}{10^6(50\text{ cm})^3}\times 50^3$$

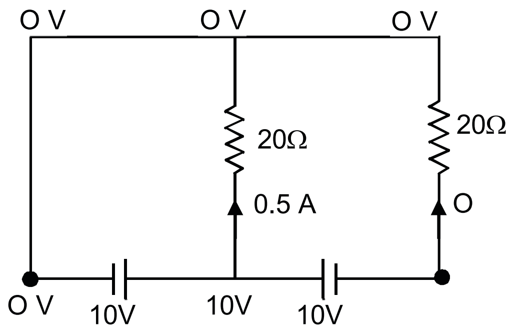
$$=\frac{64\times50\times50\times50}{100\times100\times100}\times\frac{2\text{kg}}{(50\text{ cm})^3}=8\frac{(2\text{ kg})}{(50\text{ cm}^3)}$$

73. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are :

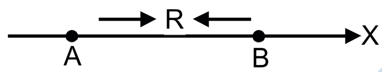


(1*) 0.5, 0 (2) 1, 2 (3) 2, 2 (4) 0, 1

Sol.

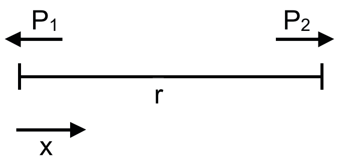


74. Two electric dipoles, A, B with respective dipole moments and are placed on the x-axis with a separation R, as shown in the figure. The distance from A at which both of them produce the same potential is :



(1) $\frac{R}{\sqrt{2}+1}$ (2) $\frac{R}{\sqrt{2}-1}$ (3) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (4*) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

Sol.



$V_1 = V_2$

$$\frac{k(2qa)}{x^2} = \frac{k(4qa)}{(R-x)^2}$$

$$R - x = \sqrt{2}x$$

$$x = \frac{R}{\sqrt{2}+1}$$

75. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to :

(1) 360 (2) 150 (3*) 200 (4) 400

Sol. In 8 seconds count rate becomes $\frac{1}{16}$ times.

∴ 4 half lives = 8s

1 half lie = 2s

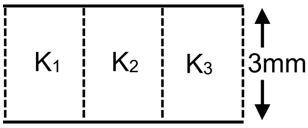
In 6s or 3 half lives, count rate $= \frac{1600}{2^3} = 200$

76. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Given : radius of earth = 6.4×10^6 m).

(1) 80 km (2*) 65 km (3) 48 km (4) 40 km

Sol. $D = \sqrt{2h_T R} + \sqrt{2h_R R}$

77. A parallel plate capacitor is of area 6 cm^2 and a separation 3mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1 = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :



(1) 36 (2) 14 (3*) 12 (4) 4

Sol. $C_{net} = C_1 + C_2 + C_3$

$$\frac{kAE_0}{d} = k_1\left(\frac{A}{3}\right)\frac{E_0}{d} + k_2\left(\frac{A}{3}\right)\frac{E_0}{d} + k_3\left(\frac{A}{3}\right)\frac{E_0}{d}$$

$$k = \frac{k_1 + k_2 + k_3}{3} = 12$$

78. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is :

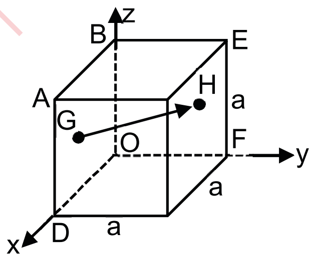
(1) $\frac{3F}{2mR}$ (2) $\frac{F}{3mR}$ (3*) $\frac{2F}{3mR}$ (4) $\frac{F}{2mR}$

Sol. F – f = ma

$$fR = \frac{1}{2}mR^2\alpha$$

a = αR

79 In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be :



(1) $\frac{1}{2}a(\hat{k}-\hat{i})$ (2) $\frac{1}{2}a(\hat{i}-\hat{k})$ (3) $\frac{1}{2}a(\hat{j}-\hat{k})$ (4*) $\frac{1}{2}a(\hat{j}-\hat{i})$

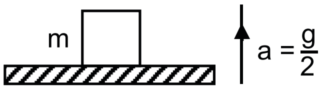
Sol. $1\left(0, \frac{a}{2}, \frac{a}{2}\right)$

$2\left(\frac{a}{2}, \frac{a}{2}, 0\right)$

$$\vec{r}_2 - \vec{r}_1 = \frac{a}{2}\hat{i} - \frac{a}{2}\hat{k}$$

Unit vector $= \frac{\hat{i} - \hat{k}}{\sqrt{2}}$

80. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in figure. Work done by normal reaction on block in time t is :



(1*) $\frac{3mg^2t^2}{8}$ (2) $\frac{mg^2t^2}{8}$ (3) 0 (4) $-\frac{mg^2t^2}{8}$

Sol. W = F s cos θ

$$= m\left(g + \frac{g}{2}\right) \times \left[\frac{1}{2} \times \frac{g}{2} \times t^2\right]$$

81. A solid metal cube of edge length 2cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is :

(1) 2mV (2*) 12mV (3) 1 mV (4) 6 mV

Sol. E = vB

v = Ed = dvB

82. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are :

(1) 400 nm, 500 nm (2) 380 nm, 500 nm (3*) 625 nm, 500 nm (4) 380 nm, 525 nm

Sol. For maxima,

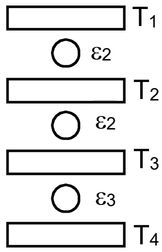
dsin θ = nλ

$\lambda = \frac{d\theta}{n}$ ($\because$ Sin θ ≈ θ when θ is small)

$= \frac{2500}{n}$nm

Take n = 4, 5 for λ = 625 nm and 500 nm.

83. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if :



(1*) $T_2 = \left(T_1^2 T_4\right)^{1/3}$; $T_3 = \left(T_1 T_4^2\right)^{1/3}$

(2) $T_2 = \left(T_1 T_4\right)^{1/2}$; $T_3 = \left(T_1^2 T_4\right)^{1/3}$

(3) $T_2 = \left(T_1^3 T_4\right)^{1/4}$; $T_3 = \left(T_1 T_4^3\right)^{1/4}$

(4) $T_2 = \left(T_1 T_4^2\right)^{1/3}$; $T_3 = \left(T_1^2 T_4\right)^{1/3}$]

Sol. $n_1 = n_2 = n_3$

$$\Rightarrow \quad 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow \quad T_2 T_3 = T_1 T_4 \text{ and } \frac{T_3^2}{T_2} = T_4$$

Solve for T_2 and T_3.

84. A 2W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is :

(1*) 20 mA (2) 63 mA (3) 0.4 mA (4) 100 mA

Sol. By colour coding,

$R = 50 \times 10^2 \Omega$

$I^2 R = P$

$$I = \sqrt{\frac{P}{R}} = 20 \text{ mA}$$

85. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is :

(1) 1 : 4 (2) 1 : 2 (3) 1 : 8 (4*) 1 : 16]

Sol. Area $\propto \pi(\text{Range})^2 \propto v^4$

$$\therefore \quad \frac{A_1}{A_2} = \left(\frac{1}{2}\right)^4$$

86. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as :

(1) $3\mu_2 - 2\mu_1 = 1$ (2) $2\mu_2 - \mu_1 = 1$ (3) $\mu_1 + \mu_2 = 3$ (4*) $2\mu_1 - \mu_2 = 1$

Sol. $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right); \frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right)$

$\frac{1}{f_1} = \frac{(\mu_1 - 1)}{1}; \frac{1}{f_2} = -\left(\frac{(\mu_2 - 1)}{R}\right)$

$f_2 = 2f_1$; $\frac{1}{f_1} = \frac{2}{f_2}$

$\frac{\mu_1 - 1}{R} = \frac{-2(\mu_2 - 1)}{R}$

$\mu_1 + 2\mu_3 = 3$

87. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ($g = 10\ ms^{-2}$)

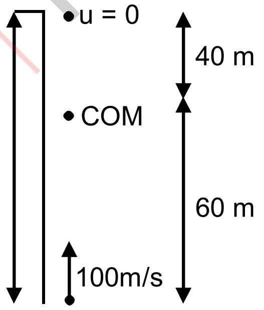
(1) 20 m (2) 30 m (3) 10 m (4*) 40 m

Sol. T_{cm} From ground $= \frac{0.03 \times 100}{0.05} = 60$ m

$V_{cm} = \frac{0.02 \times 100}{0.05} = 40$ m

$H = \frac{V_{cm}^2}{2g} = \frac{40 \times 40}{20} = 80$m

Height above building = 80 – 40 = 40 m



88. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is:

(1) 18/17 (2) 20/19 (3) 21/20 (4*) 19/18

Sol. $f_1 = f_0\left(\frac{340}{340 - 17}\right)$; $f_2 = f_0\left(\frac{340}{340 - 34}\right)$

$\frac{f_1}{f_2} = \frac{306}{323}$ or $\frac{18 \times 17}{19 \times 17}$ or $\frac{18}{19}$

89. A magnet of total magnetic moment $10^{-2}\,\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\,\hat{i}(\cos\omega t)$ where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is:

(1) 0.007 J (2) 0.01 J (3) 0.014 J (4) 0.028 J

Sol. Work done by external agent = $U_f - U_i$

= 2 MB

90. A string of length 1m and mass 5g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to:

(1) 16.6 cm (2*) 20.0 cm (3) 33.3 cm (4) 10.0 cm

Sol. $f = \frac{n}{2\ell}\sqrt{\frac{T}{m}}$

On solving, n = 5,

5 loops are formed in 1 m.

$\therefore$ Separation between successive nodes $= \frac{1}{5}\text{m} = 20$ cm